8th International Olympiad on Astronomy and Astrophysics

Suceava - Gura Humorului - August 2014

Instructions

1. In your folder you will find the following items:
    a. Answer sheets
    b. Rough work sheets
    c. The envelope with the problems The solutions of the problems should be written only on the answer sheets you receive. **PLEASE WRITE ONLY ON THE PRINTED SIDE OF THE PAPER SHEET. DON'T USE THE REVERSE SIDE**. The evaluator will not take into account what is written on the reverse side of the answer sheet.
2. The rough worksheets are for your own use for doing calculations, write some numbers etc. BEWARE: These sheets are not taken into account for the evaluation. At the end of the test they will be collected separately. Everything you consider as part of the solutions should be written on the answer sheets.
3. Each problem should be started on a separate answer sheet.
4. On each answer sheet please fill in the designated boxes as follows:
    a. In the „PROBLEM NO." box write down only the number of the problem: i.e. 1 – 12 for each short problems, 13 – 15 for each long problems. Each sheet containing the solutions of a certain problem, should have in the box the number of the problem;
    b. In „Student ID" – fill in your ID that you will find on your envelope, consisting of 3 leters and 2 digits.
    c. In the „page no." box you will fill in the number of page, starting from 1. We advise you to fill this boxes after you finish the test
5. We don't understand your language, but the language of Mathematics is universal, so, please, use as many mathematical expressions as you think that may help the evaluator to better understand your solutions. If you want to explain something in words we kindly ask you to use short phrases(if possible in English).
6. Use the pen you find out on the desk. It is advisable to use a pencil for the sketches.
7. At the end of the test:
    d. Don't forget to put your papers in order.
    e. Put the answer sheets in folder 1. Please verify that all the pages contain your ID, correct numbering of the problems and all pages are in the right order and numbered. This will help us to understanding your solutions.
    f. Verify with the assistant the correct number of answer sheets used and fill in this number on the cover of the folder and sign it.
    g. Put the draft papers in the designated folder. Put the test papers back in the envelope.
    h. Go to swim

GOOD LUCK !

Problem 1. Lagrange Points

The *Lagrange* points are the five positions in an orbital configuration (assume circular orbits), where a small object is stationary relative to two big bodies, only gravitationally interacting with them- for example, an artificial satellite relative to Earth and Moon, or relative to Earth and Sun. In the **Figure 1** are sketched two possible locations of Lagrange points L_3 relative to relative to the Earth – Sun system . Find out which of the two locations L_3^1 and L_3^2 could be the real Lagrange point relative to the system Earth – Sun; show the reason for your

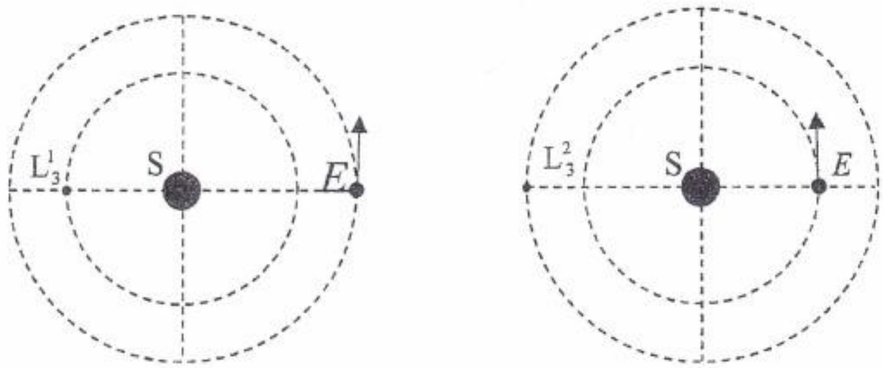

Figure 1A

Figure 1B

answer with appropriate equations and calculate the difference between one AU and Sun - L_3 distance . You know the following data: the Earth - Sun distance $d_{ES} = 14.96 \cdot 10^7$ km and the Earth – Sun mass ratio $M_E / M_S = 1/332946$

Problem 2. Sun gravitational catastrophe!

In a gravitational catastrophe, the mass of the Sun mass decrease instantly to half of its actual value. If you consider that the actual Earth orbit is elliptical, its orbital period is $T_0 = 1\ year$ and the eccentricity of the Earth orbit is $e_0 = 0.0167$.

Find the period of the Earth`s orbital motion, after the gravitational catastrophe, if it occurs on: a) 3rd of July (aphelion) b) 3rd of January.

Problem 3. Cosmic radiation

During studies concerning cosmic radiation, a neutral unstable particle – the π^0 meson was identified. The rest-mass of meson π^0 is much larger than the rest-mass of the electron. The studies reveal that during its flight, the meson π^0 disintegrates into 2 photons. In a particular case, one of the created photons has the maximum possible energy E_{max} and, consequently, the other one has the minimum possible energy E_{min}.

Find an expression for the initial velocity of the meson π^0, as a function of E_{max} and E_{min}. You may use as known **c** - the speed of light and the relation between the energy and momentum of any relativistic particles $E^2 = p^2c^2 + m_0^2c^4$

Problem 4. Sandra Bullock And George Cloony

An astronaut, with mass $M = 100$ kg, gets out of the space ship for a repairing mission. He has to repair a satellite at rest relative to the space ship, at about $d = 90\,\mathrm{m}$ away from it. After he finishes his job, he realizes that the systems designed to assure his come-back to shuttle are broken. He also observes that he has air only for 3 minutes. He also notices that he possessed a sealed cylindrical can (base section $S = 30\,\mathrm{cm}^2$) firmly attached to his/her glove, with $m = 200$ g of ice inside. The can is not completely filled with ice.

Determine if the astronaut is able to return safely to the shuttle, before his air reserve is empty, if he manages to open the can in correct direction. Briefly explain your calculations. Note that he cannot throw away anything of its equipment, or touch the satellite.

You may use the following data: $T = 272\,\mathrm{K}$ / the temperature of the ice in the can, $p_s = 550$ Pa - the pressure of the saturated water vapors at the temperature $T = 272\,\mathrm{K}$; $R = 8300$ J/(kmol·K) - the universal gas constant; $\mu = 18$ kg/kmol - the molar mass of the water.

Problem 5. The life –time of a main sequence star

The plot of the function $\log(L/L_S) = f(\log(M/M_S))$ for data collected from a number of stars is represented in figure 2. L and M are the luminosity and the mass of a star respectively and L_S and M_S the luminosity and the mass of the Sun respectively.

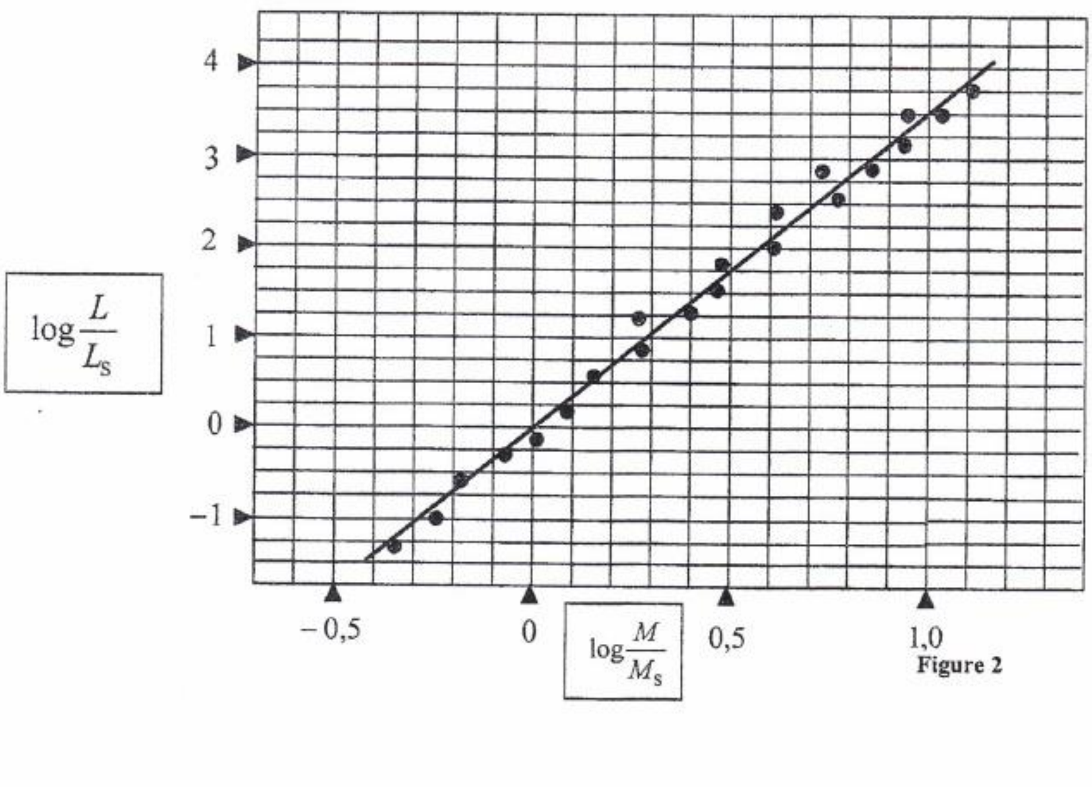

Figure 2

Find an expression for the main sequence life- time for a main sequence star from Hertzprung – Russell diagram, as a function of mass fraction converted to energy η and mass ratio to the solar mass γ , Use the following assumptions: the time spent by Sun in the same Main Sequence is τ_S , for each star the mass fraction which changed into energy is η , the percent of the mass of Sun which changes into energy is η_S , the mass of each star is expressed as $\gamma = \frac{M}{M_S}$ and assume that luminosity of the star remains constant, during its main sequence life time.

Problem 6. The effective temperature on the surface of a star

From the radiation emitted by a star, two radiations with wavelength values in a narrow range $\Delta\lambda << \lambda$ are studied, i.e. the wavelength have values between λ and $\lambda + \Delta\lambda$. According to Planck's relationship (for an absolute black body), the following relation defines, the energy emitted by star in unit time, through a unit area of its surface, per unit wavelength interval:

$$r = \frac{2\pi hc^2}{\lambda^5\left(e^{\frac{hc}{k\lambda T}} - 1\right)}.$$

The spectral intensities of the radiation with wavelengths λ_1 and respectively λ_2 , both within the range $\Delta\lambda$ measured on Earth are $I_1(\lambda_1)$ and $I_2(\lambda_2)$ respectively.

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Find out the relation between wavelength λ_1 and λ_2, if $I_1(\lambda_1) = 2I_2(\lambda_2)$, when $hc << \lambda kT$.

Here: h – Planck's constant; k – Boltzmann's constant; c – speed of light in vacuum. $e^x \approx 1 + x$ *if* $x << 1$

✓ Problem 7. Pressure of light

For an observer on Earth the pressure of the radiation emitted by Sun is $p_{\mathrm{rad,S}}$ and the pressure of the radiations emitted by a star Σ is $p_{\mathrm{rad},\Sigma}$.

Calculate the visual apparent magnitude of the star Σ if the apparent visual magnitude of the Sun is m_S .
The following assumption may be useful for solving the problem:

Generally, the pressure of the electromagnetic radiation in vacuum is equal to the volume energy density of the electromagnetic radiation $\left(p_{\text{rad}} = \frac{\Delta E}{\Delta V} \right)$.

The following data are known: M_S - the mass of the Sun, R_S - the radius of Sun, G - universal gravitational constant; σ Stefan - Boltzmann's constant ; c – speed of light in vacuum

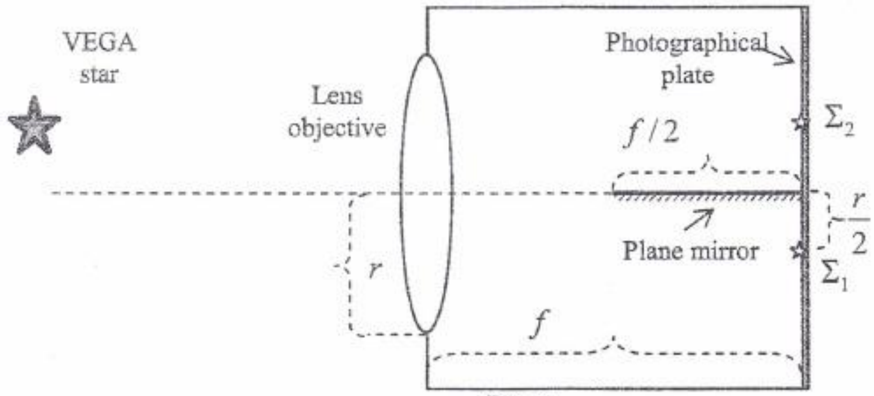
Problem 8. Space – ship orbiting the Sun

A spherical space –ship orbits the Sun on a circular orbit, and spin around an axis of rotation that is perpendicular to the orbital plane of the space-ship. The temperature on the exterior surface of the ship is T_N. Assume the space -ship is a perfect black body and there is no activity inside it .

Find out the apparent magnitude of the Sun and the angular diameter of the Sun as seen by the astronaut on board of the space – ship. The following values are known:, T_S - the effective temperature of the Sun; R_S - the radius of the Sun; d_0 - the Earth –Sun distance; m_0 - apparent magnitude of Sun measured from Earth; R_N - the radius of the space –ship.

Problem 9. The Vega star in the mirror

Inside a camera a plane mirror is placed along the optical axis of the objective (as shown in figure). The length of the mirror is half the focal length of the objective. A photographic plate is placed at the focal plane of the camera. Two images with different brightness are captured on the photographic plate (as shown in figure). The star Vega is not on the optical axis of the lens. The distance between the optical axis and the image Σ_1 is $\frac{r}{2}$ .Find the difference between the apparent photographical magnitudes of the two images of the star Vega.



Figure

Problem 10. Stars with Romanian names

Two Romanian astronomers Ovidiu Tercu and Alex Dumitriu from Galati Romania, recently discovered two variable stars. The galactic coordinates of the two stars are: Galati V 1 $\left(l_1 = 114.371^\circ; b_1 = -11.35^\circ\right)$ and Galati V 2 $\left(l_2 = 113.266^\circ; b_2 = -16.177^\circ\right)$.

Estimate the angular distance between the stars Galati V1 and Galati V2

Problem 11. Apparent magnitude of the Moon

The apparent magnitude of the Moon as seen from the Sun is $M_M = 0.25^{\mathrm{m}}$

Calculate the values of the apparent magnitudes of the Moon (as seen from the Earth) corresponding to the following Moon – phases : full-moon and the first quarter. Assume: the Moon – Earth distance - $d_{ME} = 385000\,\mathrm{km}$, the Earth – Sun distance - $d_{ES} = 1\,\mathrm{A}U$, the Moon –Sun distance, $d_{MS} = 1\,\mathrm{A}U$. For terrestrial observers, following phase factor must be used to correct the lunar brightness for curvature of lunar surface and phase of the moon.

$p(\Psi) = \frac{2}{3}\cdot\left[\left(1-\frac{\Psi}{\pi}\right)\cos\Psi + \frac{1}{\pi}\sin\Psi\right]$, where Ψ is the phase angle.

Problem 12. Absolute magnitude of a cepheid

The cepheids are variable stars, whose luminosities vary due to stellar pulsations. The period of the oscillations of a cepheid star is:

$$P = 2\pi R\sqrt{\frac{R}{GM}},$$

where: R – the mean radius of the cepheid; M – the mass of the cepheid (remains constant during oscillation), you can assume that the temperature is constant during the pulsation;

Express the mean absolute magnitude of the cepheid M_{cep}, in the following form:

$$M_{cef} = -2{,}5^{\mathrm{m}}\cdot\log k - \left(\frac{10}{3}\right)^{\mathrm{m}}\cdot\log P,$$

where P is the period of cepheid's pulsation.